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REFLECTION MATRIX FOR OPTICAL RESONATORS IN FEL OSCILLATORS

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The transformations of Gaussian radiation beams caused by reflection off mirrors is an important issue for FELs operating as oscillators. The reflected radiation from a single incident Gaussian mode will contain other modes due to the finite mirror size, the deflection of the beam and mismatches in the curvature. A method for analytic computation of the reflection matrix is developed by taking the convolution of the source function at the surface of the mirror with the paraxial propagator. The mirror surface that reflects spherical incoming wavefronts into spherical outgoing is found to be a paraboloid. Integral expressions for the reflection coefficients R_{pq}^{mn} for any incoming mode u_{mn} into the outgoing u_{pq} are obtained as functions of the deflection angle ϕ, the reflected beam spot size W_o and the mirror size. The coefficient R_{00}^{00} for the lowest-to-lowest mode reflection is determined analytically. The spot size W_o can then be selected, depending on the mirror size, to maximize R_{00}^{00}. The ratio of the mirror size to the spot size is the dominant factor determining the reflection coefficient. The effects of deflecting the light beam enter as small corrections, of first order in the diffraction angle $\theta_d \ll 1$.

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I. INTRODUCTION

Free Electron Lasers (FEL) operating as oscillators[1-7] require the trapping of light pulses between systems of mirrors (resonators).[8-10] These pulses are repeatedly amplified via synchronous interaction with electron pulses passing through the wiggler. The simplest oscillator configuration is that of an open resonator with two opposed identical mirrors. The radiation vector potential for this arrangement is expressed in terms of the free space eigenmodes $A_{mn}(r) = A_{mn}(r)e_{mn}$ of the paraxial wave equation[11], where e_{mn} is the polarization vector and

$$A_{mn}(r) = \frac{u_{mn}(r;W)}{\left(1 + \frac{z^2}{b^2}\right)^{1/2}} e^{i\left[kz + \frac{k(x^2 + y^2)}{2R(z)}\right]} e^{i\delta_{mn}(z)} . \tag{1}$$

The exponent $\Phi(r) \equiv k[z + (x^2+y^2)/2R(z)]$ in (1) contains the rapidly varying phase on the wavelength scale $\lambda = 2\pi/k$. The constant phase wavefronts are spherical of curvature $1/R(z) = z/(z^2 + b^2)$. The spot size of the radiation envelope is $W(z) = w (1 + z^2/b^2)^{1/2}$, where $w = (2b/k)^{1/2}$ is the waist, and the distance z is measured from the position of the waist. The amplitude squared of the mode drops by 1/2 over a distance equal to the Rayleigh length b (also known as confocal parameter). Most of the radiation is confined within a cone parametrized by the diffraction angle $\theta_d = W/z \simeq (\lambda/b\pi)^{1/2}$. The amplitude profile $u_{mn}(r;W)$ contains the transverse spatial variation, equivalent to a small $k_\perp$, perpendicular to the z-direction. Higher modes correspond to a larger effective $k_\perp$, responsible for the phase shift $\exp[i\delta_{mn}(z)]$. For a given k the mode is completely defined by the two independent parameters R and w (or any combination of two out of the four quantities R, w, z and b).

The functions $u_{mn}(r;W)$ depend on the elected coordinate system. In

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rectangular coordinates (x,y,z) they are given by

$$u_{mn}(x,y;W) = a_{mn}H_m\left(\frac{\sqrt{2}x}{W}\right) H_n\left(\frac{\sqrt{2}y}{W}\right) e^{-\frac{x^2+y^2}{W^2}} , \qquad (2)$$

where H_m, H_n are the Hermite polynomials, $a_{mn} = (2/W^2)^{1/2}(\pi\ 2^{m+n}\ m!\ n!)^{-1/2}$ is the normalization factor, setting the total electromagnetic flux carried by the mode equal to unity, and $\delta_{mn}(z) = (m + n + 1) \tan^{-1}(z/b)$ is the slow phase. In cylindrical coordinates (r,θ,z), where $\tan\theta = x/y$, $r = (x^2+y^2)^{1/2}$, $u_m^p(r,\theta;W)$ take the form

$$u_m^{\pm p}(r,\theta;W) = a_m^p \begin{pmatrix} \cos p\theta \\ \sin p\theta \end{pmatrix} \left(\frac{\sqrt{2}r}{W}\right)^p L_m^p\left(\frac{2r^2}{W^2}\right) e^{-\frac{1}{2}\frac{2r^2}{W^2}} , \qquad (3)$$

where $+p$ $(-p)$ signifies cosine (sine) poloidal dependence, L_m^p are the Laguerre polynomials, $a_m^p = (1/2\pi W^2)^{1/2} [m!/(m + p)!]^{-1/2}$ and $\delta_m^p(z) = (2m + p + 1) \tan^{-1}(z/b)$.

The electron beam is an *optically* active medium that alters the characteristic parameters of the radiation after each passage. During the build-up period the modal content and the structure of the light pulses inside the oscillator will change. A numerical method has been developed recently optimizing the representation for the amplified radiation. In the source dependent expansion[12,13] the waist size and the curvature of the elected modal basis is tailored according to the driving source term. That minimizes the number of modes required to describe the light beam. In general, the curvature and waist size of these modes does not match the curvature and waist of the vacuum eigenmodes for the resonator. Therefore the transfer matrix for a given mirror must be known for arbitrary incoming modes. This need stems from computational as well as physical reasons. The knowledge of the cavity reflection matrix **R**, together with the gain

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matrix **G** through the wiggler, is necessary in determining the potential for steady state operation.

During high power operation, grazing mirror incidence may be necessary to avoid exceeding the dielectric breakdown limit for the reflection surface. Also, in case of a high per-pass gain with optical guiding, the waist for the reflected radiation would be much larger than the incoming. In two-mirror resonators the reflected radiation could hit the wiggler. Therefore, ring resonators, including three or more mirrors, must be employed for the deflection and recirculation of the light pulses. The study of the reflection matrix must be extended to include the effects of deflecting the light beam, in addition to finite mirror size and curvature mismatches.

The amplitude profile of the incoming radiation will be modified by reflection. A single incident mode $A_{mn}(r_i)$ will, in general, be partially reflected into different modes $A_{pq}(r_o)$ where $(m,n) \neq (p,q)$. This is caused by the deflection of the light beam, the finite size of the mirror and the curvature mismatches. Reflection into other modes will affect the interaction between the electron beam and the radiation in a number of ways. First the rms radius of the light beam will change, affecting the matching beam condition. Second, the light pulse will spread axially because of dispersion among different modes, since the phase velocity depends on the modal number (m,n). Third, different phase advances during reflection among the various modes may render these modes out of phase after a number of bouncings off the resonator. For the above reasons the fraction of radiation scattered into other modes will contribute to the losses in FEL oscillators.

II. OUTLINE OF THE METHOD

The geometry of the reflection is illustrated in Fig. 1. The subscripts i and o denote the coordinate system used for expressing incoming and outgoing modes. r_i is defined with the $\hat{z}_i$ axis along the direction of incidence and r_o has the $\hat{z}_o$ axis along the direction of reflection. The origins are displaced from the mirror center by l_i and l_o respectively, where l_i is the distance of the minimum waist w_i for the incoming radiation and l_o is the distance of the minimum waist w_o for the reflected mode. A third coordinate system r_s with the origin at the mirror center and $\hat{z}_s$ aligned with the mirror axis will be useful in the computations. Underlined quantities $\underline{r}_i$, $\underline{r}_o$ and $\underline{r}_s$ stand for the mirror surface coordinates in each reference frame. The transformations among the various frames are defined by

$$x_i = x_s \cos \frac{\phi}{2} - z_s \sin \frac{\phi}{2} , \qquad\qquad x_o = x_s \cos \frac{\phi}{2} + z_s \sin \frac{\phi}{2} ,$$

$$y_i = y_s , \qquad\qquad (4a) \qquad y_o = y_s , \qquad\qquad (4b)$$

$$z_i = z_s \cos \frac{\phi}{2} + x_s \sin \frac{\phi}{2} + l_i , \qquad z_o = z_s \cos \frac{\phi}{2} - x_s \sin \frac{\phi}{2} + l_o .$$

We are interested in cases when the reflected radiation remains focused along some direction $\hat{z}_o$ making an angle ϕ with the incoming $\hat{z}_i$. Then the reflected vector potential will also be expandable in free space eigenmodes $A_{pq}(r_o)$ of the paraxial equation in the new direction. The mirror surface generating focused reflection in the desired direction can not be arbitrary but must be appropriately defined. The angle of deflection ϕ will enter the equation defining the mirror surface. The other surface parameter, namely the curvature $1/R_m$, is a free parameter. It determines the curvature $1/R_o$ for the outgoing modes given the curvature $1/R_i$ of the incoming modes. In case of reflection by an arbitrary mirror

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surface, the scattered radiation cannot, in general, be covered by the paraxial modes that do not form a complete set in three dimensions.

We consider incoming radiation of given curvature and of arbitrary amplitude profile $A^i(r_i)$, consisting of various modes (m,n) with the same $R_i(z)$. If both incident and reflected radiation are expanded into eigenmodes,

$$A^i(r_i) = \sum_{m,n} c^i_{mn} A_{mn}(r_i) \;,$$

$$A^o(r_o) = \sum_{p,q} c^o_{pq} A_{pq}(r_o) \;,$$

(5)

the relation among the incident and reflected expansion coefficients c^i_{mn}, c^o_{pq} is formulated by

$$c^o = R \, c^i \;,$$

(6a)

or

$$c^o_{pq} = \sum_{m,n} R^{mn}_{pq} \, c^i_{mn} \;,$$

(6b)

where R^{mn}_{pq} are the elements of the reflection matrix R.

We examine the case when the mirror dimensions ρ are much larger than the wavelength λ, $\lambda \ll \rho$ (otherwise diffraction rather than reflection would prevail). We also assume that the angle ζ subtended by the mirror $\zeta = \rho/R_m$, where R_m parametrizes the radius of curvature, is small, of the order of the diffraction angle θ_d, $\zeta \sim \theta_d \sim \varepsilon$. The j-th component of the reflected vector potential at distance $|r_o - \underline{r}_o| \gg \lambda$ from the mirror surface S is then given by

$$A^o_{(\nu)}(r_o) = -\frac{ik}{2\pi} \iint_S ds \, \frac{e^{ik|r_o - \underline{r}_o|}}{|r_o - \underline{r}_o|} A^s_{(\nu)}(\underline{r}_o) \, (\hat{n} \cdot \hat{\Delta r}).$$

(7)

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In Eq. (7) $\hat{n} \cdot \Delta \hat{r}$ is the obliqueness factor where $\Delta \hat{r} = (r_o - \underline{r}_o)/|r_o - \underline{r}_o|$ and $\hat{n}$ is the normal unit vector to the reflecting surface. The surface element ds is given by $ds = \delta[z_o - f(x_o, y_o)] dx_o dy_o dz_o$ where $z_o = f(x_o, y_o)$ is the surface equation. Equation (7) is the convolution of a source term $A^S(\underline{r}_o)$ at the mirror surface with the propagator $\exp(ik|r_o - \underline{r}_o|)/|r_o - \underline{r}_o|$, i.e., a superposition of spherical waves originating at S. The source term $A^S(\underline{r}_o)$ is specified from the incoming vector potential $A^i(\underline{r}_i)$ through the boundary conditions and the coordinate transformations (4). We will assume a perfectly conducting surface, where the incident and reflected fields are related by

$$A^S = -A^i + 2\,(\hat{n} \cdot A^i)\,\hat{n}\,, \tag{8a}$$

and $\hat{n}$ is the normal unit vector to the reflecting surface. When the solid angle subtended by the mirror is small, the last term in (8a) is very small and the boundary condition becomes,

$$A^S_{(\nu)} = -A^i_{(\nu)}\,. \tag{8b}$$

Relation (8b) corresponds to a phase shift by π during reflection. It is independent of the wave polarization, thus the subscript (ν) is dropped. Cross polarization effects, due to the last term in (8a) are discussed in Ref. 10. Most of the computations will be performed on the mirror surface. To simplify the notation from now on, we drop the bar (_) under the mirror coordinates $\underline{r}$. Subscripted quantities such as r_i, r_o, r_s will signify the mirror surface in each reference frame. Unsubscripted coordinates will denote the observation point in the reflected radiation frame of reference.

We seek cases when the reflected radiation propagates focused along z_o, contained within a cross section of dimensions $x, y \ll z - z_o$. The

expansion $|r-r_0| \simeq (z-z_0) \{1 + [(x-x_0)^2 + (y-y_0)^2]/2(z-z_0)^2\}$ replaces the full propagator inside (7) with the paraxial propagator U_{-k} in that direction,

$$U_{-k} (r, r_0) = \frac{ik}{2\pi} \frac{e^{-ik(z-z_0)}}{z-z_0} e^{-ik \frac{(x-x_0)^2 + (y-y_0)^2}{2(z-z_0)}} . \qquad (9)$$

It is known that the profile of a given eigenmode $A_{mn}(x_0,y_0,z_0)$ at z_0 is generated by the propagator $U_k(r,r_0)$ acting on the mode $A_{mn}(x,y,0)$ at $z=0$. The inverse propagator $U_{-k}(r,r_0)$ therefore reproduces $A_{mn}(x,y,0)$ from $A_{mn}(x_0,y_0,z_0)$. This suggests multiplying and dividing the integrand inside (7) by $\exp[i\Phi(r_0)] / [1 + z_0^2/b_0^2]^{1/2}$, recasting (7) in the form,

$$A^0(r) = \iint ds \, e^{i\Delta(r_0)} S(r_0) \, e^{i\Phi_0(r_0)} U_{-k}(r,r_0) , \qquad (10)$$

where the source term $S(r_0)$ is,

$$S(r_0) = A^i[r_i(r_0)] \, (\hat{n} \cdot \hat{\Delta r}) \left[1 + \frac{z_0^2(r_0)}{b_0^2}\right]^{1/2} , \qquad (11)$$

and the phase $\Delta(r_0)$ is given, in outgoing coordinates r_0, by

$$\Delta(r_0) = k \left[z_i(r_0) + z_0 + \frac{x_i^2(r_0) + y_i^2(r_0)}{2R_i(r_0)} + \frac{x_0^2 + y_0^2}{2R_0(r_0)} \right]. \qquad (12)$$

The expression for $\Delta(r_0)$ depends on the angle ϕ through the transformations between the incoming and the outgoing coordinates, Eqs. (4). Expression (10) is the approximation of the exact solution (7) to order

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$[(x-x_o)^2 + (y-y_o)^2]/2(z-z_o)^2 \sim \varepsilon^2$. It is valid provided that the surface S produces focused reflection along the desired direction. Otherwise, the paraxial limit will fail to encompass all the radiation contained in the original expression (7).

The term $\exp[i\Delta(r_o)]$ is varying rapidly, on the scale of the wavelength λ. Therefore, its convolution with the slowly varying source term over an arbitrary surface will be vanishingly small. In general, this corresponds to radiation scattering where only a small fraction of the incoming radiation is reflected along the considered direction ϕ. The integral (10) will be finite only when it is possible to satisfy the condition $\Delta(r_o) \simeq$ constant over some surface S. If, in addition, S is much larger than λ, expression (10) will be finite only within a narrow angle $\delta\phi$ around ϕ. This guarantees that the reflected radiation remains focused along that direction. Therefore, a condition that the exact reflected radiation (7) be fully covered by the paraxial limit (10) is that

$$\Delta(r_o) = \text{const.}, \tag{13}$$

along the surface S. Accordingly, the optical path is the same along the various rays connecting an incoming wavefront with its mirror image (reflected) wavefront.

Requirement (13) defines the appropriate mirror surface $z_o = f_o(x_o, y_o; \phi)$ for reflection in the elected direction. Expressing all quantities inside (13) in the mirror coordinate frame, applying the transformations (4) and using the scaling $x_s/R_m \sim y_s/R_m \sim \varepsilon \ll 1$, $z_s/R_m \sim \varepsilon^2$ we obtain from (13)

$$z_s = - \frac{1}{2R_m \cos \frac{\phi}{2}} \left[x_s^2 \cos^2 \frac{\phi}{2} + y_s^2 \right], \tag{14a}$$

where

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$$\frac{1}{R_m} = \frac{1}{2R_o} + \frac{1}{2R_i}. \tag{14b}$$

Equation (14a) is the analytic expression for a paraboloid surface. R_m parametrizes the mirror curvature, being positive or negative for a convex or concave mirror respectively. The surface is reflection symmetric with $(zx)_s$ and $(zy)_s$ as the symmetry planes; there is no rotational symmetry around $\hat{z}_s$. Surface (14a) can also be approximated, to second order in $(x_s/R_m)^2$, $(y_s/R_m)^2$ by hyperboloids or ellipsoids defined by

$$\left(z_s - R_m \cos \frac{\phi}{2}\right)^2 - x_s^2 \cos^2 \frac{\phi}{2} - y_s^2 = R_m^2 \cos^2 \frac{\phi}{2}, \tag{15a}$$

$$\left(z_s + R_m \cos \frac{\phi}{2}\right)^2 + x_s^2 \cos^2 \frac{\phi}{2} + y_s^2 = R_m^2 \cos^2 \frac{\phi}{2}. \tag{15b}$$

All the surfaces become spherical in the limit of perpendicular incidence $\phi = 0$, and plane mirrors when $R_m \rightarrow \infty$. Relation (14b) defines the curvature of the reflected modes from the incoming mode curvature and the curvature of the mirror.

Switching Eq. (12) into the mirror-aligned coordinates r_s through Eqs. (4), and using the surface constraints (14), it follows that

$$\Delta(r_s) = \text{const.} + 0\left[k\rho \left(\frac{\rho}{R_m}\right)^2 \right],$$

where ρ parametrizes the mirror size. A more complicated surface equation (higher than quadratic in x, y, z) is required to improve the constancy to a higher order. Since $k\rho \gg 1$, the approximation $\Delta(r_s) = \text{const.}$ is satisfactory for a first order expansion of the reflection matrix in powers of ρ/R_m, as long as $\rho/R_m \leq (k\rho)^{-1}$. In case $\rho/R_m > (k\rho)^{-1}$, the slow variation of $\Delta(x_s, y_s)$ over S must be included. That introduces an additional contribution in the reflection matrix, known as spherical aberation.

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III. COMPUTATION OF THE REFLECTION MATRIX

The reflected radiation is expressed by

$$A^O(r) = \int\int_S dx_0 dy_0 \; \sigma(x_0,y_0) \; e^{-i\Phi_0(x_0,y_0)} \; U_{-k}(r,r_0) \; , \qquad (16)$$

where $\sigma(x_0,y_0) \equiv S[x_0,y_0,z_0(x_0,y_0)]$. Expanding the source $\sigma(x_0,y_0)$ in terms of $u_{mn}(x_0,y_0)$,

$$\sigma(x_0,y_0) = \sum_{m,n} R^{mn} u_{mn}(x_0,y_0;W_0) \; . \qquad (17)$$

and exploiting the property of the inverse propagator U_{-k}, the reflected vector potential $A^O(r)$ at $z = 0$ becomes

$$A^O(x,y,0) = \sum_{m,n} R^{mn} u_{mn}(x,y;W_0) \; , \qquad (18)$$

where $W_0(z) = w_0 (1 + z^2/b_0^2)^{1/2}$, $w_0 = (2b_0/k)^{1/2}$. Expression (18) is a complete decomposition of the reflected radiation into paraxial eigenmodes for incident radiation of arbitrary profile.

According to the definition (6b), the R^{mn}_{pq} element of the reflection matrix R is obtained from the source term $\sigma_{pq}(x_0,y_0)$ inside (16) generated by a single incident eigenmode $A_{pq}[r_i(r_0)]$. The integration is performed in the mirror-aligned coordinates, taking advantage of the existing symmetries. The coordinates r_i and r_0, defining the incoming and outgoing wave functions, become explicit functions of x_s, y_s through the transformations (4). The surface equation (14a) is used to express z_s in terms of (x_s,y_s). The mirror boundary

$$x_s^2 \cos^2 \frac{\phi}{2} + y_s^2 = \rho^2 \tag{19}$$

is defined by the intersection of the infinite surface (14a) with the plane $z_s = \text{const} = 2 \; \rho^2 \cos^2(\phi/2)/ \; R_m$. After the above manipulations the reflection matrix elements take the form

$$R_{pq}^{mn} = \iint_S dx_s dy_s \frac{\bar{u}_{mn}(x_s,y_s) \; \bar{u}_{pq}(x_s,y_s)}{\left[1 + \frac{l_o^2}{b_o^2}\right]^{1/2}} \left[\frac{1 + \frac{z_o^2(x_s,y_s)}{b_o^2}}{1 + \frac{z_i^2(x_s,y_s)}{b_i^2}}\right]^{1/2} e^{i\Delta(x_s,y_s)} \tag{20}$$

$$\times \; e^{i\delta_{pq}^i(x_s,y_s) - i\delta_{mn}^o(x_s,y_s)} \left[\cos \frac{\phi}{2} \left(1 - \frac{x_s}{R_m} \sin \frac{\phi}{2} - \frac{x_s^2 \sin^2 \frac{\phi}{2}}{R_m^2}\right)\right],$$

where

$$\bar{u}_{mn}(x_s,y_s) \equiv u_{mn}\left[x_o(x_s,y_s), \; y_s\right], \qquad \bar{u}_{pq}(x_s,y_s) \equiv u_{pq}\left[x_i(x_s,y_s), \; y_s\right].$$

Expression (20) is correct to order ρ^2/R_m^2.

Each representation of R is tied to the choice of the basis functions $u_{mn}(r)$. In any case R, as given by (20), depends on four parameters[10]

$$R \equiv R(\phi, \; \alpha, \; \mu; \; \xi). \tag{21}$$

ϕ is the reflection angle shown in Fig. 1. α is the ratio of the incoming to the outgoing spot size at the mirror, $\alpha = W_i(l_i)/W_o(l_o)$. $\mu = \rho/W_o$ parametrizes the mirror size compared to the radiation spot size. $\xi = W_o/R_m$ scales as the diffraction angle $\theta_d \simeq W_o/l_o$ multiplied by the curvature mismatch R_o/R_m between the mirror and the radiation wavefronts. The spot size W_o enters as a free parameter because only the curvature $1/R_o$ for the reflected modes is specified by the mirror geometry. Since many

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combinations of W_o and l_o apply to a given curvature according to paragraph Eq. (1), an additional selection rule for W_o is needed. Note that W_o does not have to match W_i. This is obvious in cases when the mirror size ρ is smaller that W_i. Each value of W_o defines a complete set of modes for the reflected radiation and an equivalent representation for R.

Parameters ϕ, α, and μ can be arbitrary. In most cases of interest, however, ξ is small, $\xi \ll 1$, of the same order as the diffraction angle θ_d. The analytic computation of the matrix elements is carried out by expanding the integral (20) in powers of ξ,

$$R = R(0) + \xi R(1) + \xi^2 R(2).\tag{22}$$

The first order expansion is performed in Ref. 10. In this paper we review some of the general properties of R and focus on the reflection of the lowest mode u_{00}.

IV. LIMITING CASES

When the mirror radius tends to infinity $(1/R_m \to 0)$, or in cases of vertical incidence on the mirror $(\phi = 0)$, the higher order corrections in the reflection matrix R disappear,

$$R = R(0)\tag{23}$$

in both representations. The nondiagonal elements in R stem from the finite mirror size only. If, in addition, the mirror size is very large, $\mu \gg 1$, it is appropriate to take $W_o = W_i$ as best representation for the reflected radiation. The $\alpha = 1$ limit yields

$$R_{pq}^{mn} = \delta_{pq}^{mn}.\tag{24}$$

Thus, in case of large curved mirror and vertical incidence, or large plane mirror and arbitrary incidence, the reflection matrix is the

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identity matrix.

The case $\alpha = 1$ is of special interest for arbitrary angle of deflection ϕ and mirror curvature $1/R$, as it will be explained in the next section. For finite mirror size $\rho \geq W_o$, ($\mu \geq 1$), there exists zeroth order nondiagonal terms inside $R(0)$. Since $R(0)$ is independent of the angle of deflection ϕ, the mirror size yields the dominant contribution to the reflection into modes different than the incoming. The effects of the deflection of the light beam enter to first order in ξ, $R(1)$, or higher.

As the mirror size becomes very large and the limits of integration are extended to infinity the orthogonality among the various modes u_{mn} becomes effective. The off-diagonal terms in $R(0)$ become comparable to the first order terms $R(1)$ roughly when $1/\mu^2 \sim \xi \sim \theta_d$. At the limit $\mu \rightarrow \infty$ all the nondiagonal elements of R are reduced to order ξ or higher,

$$R_{pq}^{mn} = \xi\, R_{pq}^{mn}(1) + O(\xi^2), \quad m \neq p,\ n \neq q, \tag{25a}$$

and the only matrix elements of zeroth order in ξ are the diagonal

$$R_{mn}^{mn} = R_{mn}^{mn}(0) + O(\xi^2). \tag{25b}$$

in both Hermite and Laguerre represesations. The lowest correction in the diagonal elements is of second order ξ^2, while the first order contribution disappears. This is consistent with flux conservation during reflection in case of a large mirror.

The superposition principle can be used to describe reflection from more complex mirror surfaces. In case of a mirror with a hole the surface integral (7) over S is expressed as $\int_S = \int_{S1} - \int_{S2}$ where S_1 is defined by the mirror surface including the hole surface, and S_2 is the surface of the hole only. The total reflection matrix R is given by $R =$

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$R(S_1) - R(S_2)$, the difference in the reflection matrices associated with mirrors S_1 and S_2 respectively. The transmission matrix **T** through a screen with an aperture of area S is given by **T** $= -$ **R**, **R** being the reflection matrix for a mirror matching the aperture S. The transmission matrix for radiation diffracted behind a finite size mirror is given by **T'** $= 1 - e^{i\pi}$ **R** where **1** is the identity matrix.

V. REFLECTION OF THE LOWEST ORDER MODE

The computation of all the truncated integrals for finite mirror surface is nontrivial. Most applications however involve the (0,0) lowest order mode as the dominant mode in both incoming and reflected radiation. The strategy here is to compute the element R_{00}^{00} of the reflection matrix first. Then the waist for the reflected modes W_o can be selected so that it maximizes R_{00}^{00}. The optimum representation condition

$$\frac{\partial R_{00}^{00}}{\partial \alpha} = 0, \tag{26}$$

puts the maximum amount of the reflected radiation in the lowest mode (a different mode and matrix element may be chosen, if desired). It is pointed out that (26) does not improve the properties of the reflected radiation. It enables one to choose the best representation in terms of minimizing the coefficients of the undesired modes for the scattered radiation. Once W_o is fixed by (26) then the exact location and size of the waist(s) for the reflected modes is determined by solving the system of equations

$$W_o = w_o \left[1 + \frac{l_o^2}{b_o^2} \right]^{1/2}, \qquad \frac{1}{R_o} = \frac{l_o}{l_o^2 + b_o^2} . \tag{27}$$

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The element R_{00}^{00} is identical in both representations since the lowest order mode u_{00} is the same in rectangular and cylindrical coordinates. Performing the integration (20) yields R_{00}^{00} to first order in ξ

$$R_{00}^{00} = \frac{2\alpha}{1+\alpha^2} \left[1 - e^{-(1+\alpha^2)\mu^2} \right] + O(\xi^2). \qquad (28)$$

Note that the first order term vanishes and the lowest correction is of second order in ξ^2. The exact dependence on the mirror size ρ is parametrized by $\mu = \rho/W_o$, while $\alpha = W_i/W_o$ parametrizes the ratio of the incoming and scattered radiation spot sizes at the mirror. The optimization condition $\partial R_{00}^{00}(0)/\partial\alpha = 0$ yields, $\alpha^2 = 1 + \exp[-(1+\alpha^2)\mu^2] \ [\alpha^4+(2\mu^2+1)\alpha^2-1]$. In case the mirror cross section is much larger than the spot size of the incoming mode, $\mu \gg 1$, $\alpha \to 1$ and the reflected spot size at the mirror matches the incoming, $W_o = W_i$.

Large mirror size is desired to maximize the total reflection coefficient. The reflection coefficient η_R is given by $\eta_R = P_o/P_i$ where the incoming flux is $P_i = |c^i|^2 = \Sigma \ |c_{pq}^i|^2$ and the outgoing flux is given by

$$P_o = |c^o|^2 = | R \cdot c^i|^2 = \sum_{mn} \sum_{pq} |R_{pq}^{mn} c_{pq}^i|^2. \qquad (29)$$

In Fig. 2 we plot η_R for the lowest order incoming mode as a function of $\mu' = \cos(\phi/2) \ \rho/W_o = \cos(\phi/2) \ \mu$. μ' parametrizes the size of the mirror surface projection into the plane perpendicular to the incoming radiation direction. The incoming radiation has a wavelength $\lambda = 1\mu$ (10^{-4}cm), waist $w_i = 10^{-2}$cm at distance $l_i = 2.5\times10^3$cm from the mirror and radius of curvature (at the mirror) $R_i = 2.5\times10^3$cm. The mirror has a radius of curvature $R_m = 2.5\times10^3$cm, yielding reflected modes of $R_o =$

2.5×10^3 (again l_0 and v_0 depend on the choice of W_0). In Fig. 3 we plot the reflection coefficients R_{pq}^{00} of the lowest order mode (0,0) into the first 25 modes (p,q) with $p \leq q \leq 5$, as a function of μ'. The deflection angle is 90° and the ratio of the spot sizes is 1. Increasing mirror size maximizes the diagonal element and minimizes scattering into other modes.

In Fig. 4 we fix the mirror size $\mu' = 2$ and the angle $\phi = 90^\circ$ and vary the spot size ratio α. The best representation, maximizing R_{00}^{00} and minimizing R_{pq}^{00} is obtained at $\alpha = 1$. However, for small mirror $\mu' = 0.66$, the maximum for R_{00}^{00} occurs at $\alpha \simeq 0.70$ (see Fig. 5). Radiation reflected off mirrors smaller than the incoming spot size is best described by outgoing modes of reduced spot size $W_0 < W_i$. Also note from Fig. 5b that for small mirror size the total power reflected into the first 25 modes never exceeds 80% of the incoming flux; even with many more modes η_R remains less than 1. In Fig. 6 the reflection coefficients R_{pq}^{00} are plotted as functions of the angle of deflection ϕ for fixed $\alpha = 1$, $\mu' = 2$. It is seen that, for sufficiently large reflecting surface and good choice of the spot size W_0, the reflection matrix is not very sensitive to ϕ. Comparison of Figs. 2 and 3 with the rest of the plots shows that the relative mirror size to the radiation spot size is the most important parameter to determine the reflection into other than the incoming modes.

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Figure 1 Reflection geometry.

Figure 2 Plot of the total reflection coefficient η_R for the lowest order mode as a function of the mirror size μ' for $\phi = 90°$ and $W_o = W_i$.

Figure 3 Plots of the reflection matrix elements of the lowest order mode (0,0) into the first 25 modes $p \leq q \leq 5$. The magnitude $|R_{pq}^{00}|$ is plotted against the relative mirror size μ'. The angle of deflection $\phi = 90°$ and $\alpha = 1$ ($W_i = W_o$).

Figure 4 Plots of (a) the reflection matrix elements $|R_{pq}^{00}|$, and (b) the reflection coefficient η_R into the first 25 modes, against the spot size ratio α for $\mu' = 2$ and $\phi = 90°$.

Figure 5 Same as in Fig. 4 for $\mu' = 0.5$.

Figure 6 Plots of the reflection matrix elements $|R_{pq}^{00}|$ against the angle of deflection ϕ for $\mu' = 2$ and $\alpha = 1$.

η_R

μ'

1.00

.90

.80

.70

.60

0.6   1.2   1.8   2.4   3.0

10^1

10^0

10^{-1}

10^{-2}

10^{-3}

10^{-4}

R^{∞}_{kl}

0.6 1.0 1.5 2.0

α

(a)

0,0

2,0

2,2

4,4

1,5

4,2

3,5

1,3

1,0

3,0

5,0

1.05

1.00

.95

.90

η_R

0.6 1.0 1.5 2.0

α

(b)

(a)

(b)